Numerical methods for optimal transport and moment measures

Quentin Mérigot
Universit Paris-Sud

Key words: Optimal transport, Brunn-Minkowski inequality, Brenier solution.

Several problems in geometry lead to equations of the form

$$g(\nabla u)) \det(\mathrm{D}^2 u) = a(u), \tag{1}$$

where $g : \mathbb{R}^d \to \mathbb{R}$ is a probability density and $a : \mathbb{R} \to \mathbb{R}$ decays sufficiently fast at infinity. For $a(t) = \exp(-t)$ this equation is related to the moment measure problem studied in [1, 3], while for $a(t) = t^{-d+2}$ this equation appears in the construction of $(d-1)$-dimensional affine hemispheres in convex geometry [2] or in the construction of Stein kernel (see Max Fathi talk on Wednesday) . As in optimal transport, one can define a Brenier solution to (1) as a convex function $u : \mathbb{R}^d \to \mathbb{R} \cup \{+\infty\}$ which satisfies $\nabla u_{\#} a(u) = \mu$, where μ is the measure on $\mathbb{R}$ with density g. When $a(t) = \exp(-t)$ or $a(t) = t^{-d+2}$, Brenier solutions to (1) maximize a concave functional similar to the one appearing in Kantorovich duality. We will show that this leads to efficient numerical methods when the measure μ is finitely supported. In the moment measure case, we will deduce the convergence of a Newton algorithm from a discrete version of the Brascamp-Lieb inequality.

This is a joint work with Bo'az Klartag (Department of Mathematics, Weizmann Institute, Israel) and Filippo Santambrogio (Universit Paris-Sud)

References

[1] Cordero-Erausquin, D., & Klartag, B. A. (2015). Moment measures. Journal of Functional Analysis, 268(12), 3834-3866.

[2] Klartag, B. A. (2015). Affine hemispheres of elliptic type. arXiv preprint arXiv:1508.00474.

[3] Santambrogio, F. (2016). Dealing with moment measures via entropy and optimal transport. Journal of Functional Analysis, 271(2), 418-436.