Asymptotic behavior of large Gaussian correlated Wishart matrices

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We consider high-dimensional Wishart matrices, associated with a rectangular random matrix X of size $n \times d$, whose entries are jointly Gaussian and correlated. Our main focus is on the case where the rows of X are independent copies of a n-dimensional stationary centered Gaussian vector of correlation function s. When s is $4/3$ integrable, we show that a proper normalization of $d^{-1}XX^T$ is close in Wasserstein distance to the corresponding Gaussian ensemble as long as d is much larger than n^3. We also investigate the case where s is the correlation function associated with the fractional Brownian noise of parameter H. This example is very rich, as it gives rise to a great variety of phenomena with very different natures, depending on how H is located with respect to $1/2$, $5/8$ and $3/4$. Notably, when $H > 3/4$, our study highlights a new probabilistic object, which we have decided to call the Rosenblatt-Wishart matrix. Our approach crucially rely on the fact that the entries of the Wishart matrices we are dealing with are double Wiener-It integrals, allowing us to make use of multivariate bounds arising from the Malliavin-Stein method and related ideas.

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